<u>11.9</u>

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad \text{CONV ON } (-1,1)$$

$$\frac{x^2}{1-x^3} = x^2 \cdot \frac{1}{1-x^3} = x^2 \sum_{n=0}^{\infty} (x^3)^n = \sum_{n=0}^{\infty} x^{3n+2}$$

SUBSTITUTE $x^3$ FOR $x$

$$= x^2 + x^5 + x^8 + \dots$$

$$\frac{x^2+1}{1-x^3} = (x^2+1) \sum_{n=0}^{\infty} x^{3n} = \sum_{n=0}^{\infty} \left(x^{3n+2} + x^{3n}\right)$$

$$= (x^2+1) + (x^5+x^3) + (x^8+x^6) + \dots$$

$$= 1 + x^2 + x^3 + x^5 + x^6 + x^8 + x^9 + x^{11} + \dots$$

COEF'S ARE ALL 1

EXCEPT $n = 1, 4, 7, 10 \dots$

COEF'S ARE ALL 0

$$= \sum_{n=0}^{\infty} c_n x^n \quad \text{WHERE } c_n = \begin{cases} 1, & \text{IF } n \in \mathbb{Z}^* \text{ AND } n \neq 3k+1 \text{ FOR ANY } k \in \mathbb{Z}^+ \\ 0, & \text{IF } n = 3k+1 \text{ FOR SOME } k \in \mathbb{Z}^+ \end{cases}$$

DOES THIS SUBSTITUTION METHOD WORK FOR $\frac{1}{1-\sqrt{x}}$ ?

$$\frac{1}{1-\sqrt{x}} = \sum_{n=0}^{\infty}\left(x^{\frac{1}{2}}\right)^n = \sum_{n=0}^{\infty} x^{\frac{1}{2}n} \leftarrow$$ THEY ARE EQUAL

$x^{\frac{1}{2}}$

BUT

$$= 1 + x^{\frac{1}{2}} + \cdots$$

THIS IS NOT A POWER SERIES

NEED FULL BLOWN TAYLOR SERIES METHOD

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$$\frac{1}{(1-x)^2} = \frac{d}{dx}\frac{1}{1-x} = \frac{d}{dx}\sum_{n=0}^{\infty} x^n = \sum_{n=0}^{\infty}\frac{d}{dx}x^n$$

$$\frac{d}{dx}(1-x)^{-1}$$

$$= -1(1-x)^{-2}(-1)$$

$$= (1-x)^{-2}$$

$$= \sum_{n=0}^{\infty} n x^{n-1}$$

IF $n=0$ $x^{n-1} = x^{-1}$

BUT $n x^{n-1} = 0x^{-1}$
$= 0$

$$= \sum_{n=1}^{\infty} n x^{n-1}$$

$$= \underset{n=0}{1x^0} + \underset{n=1}{2x^1} + \underset{n=2}{3x^2} + \underset{n=3}{4x^3} + \cdots$$

$$= \sum_{n=0}^{\infty}(n+1)x^n$$

INDEX SHIFT

$$\sum_{n=1}^{\infty} n x^{n-1} = \sum_{k=0}^{\infty} (k+1) x^{k} = \sum_{n=0}^{\infty} (n+1) x^{n}$$

LET $k = n-1$ — $n=\infty \longrightarrow k=\infty$; $n=1 \longrightarrow k=0$

$n = k+1$

ALWAYS ONLY $k = n \pm c$

OR

$$\sum_{n=1}^{\infty} n x^{n-1} = \sum_{n=0}^{\infty} (n+1) x^{n} = \sum_{n=-4}^{\infty} (n+5) x^{n+4}$$

n GOES UP BY 4 INSIDE

n GOES DOWN BY 4 ON LIMITS

n GOES DOWN BY 1 ON LIMITS OF $\Sigma$.

n GOES UP BY 1 INSIDE $\Sigma$

FIRST TERMS

$1x^0 + 2x^1 + 3x^2 \qquad 1x^0 + 2x^1 + 3x^2 \qquad 1x^0 + 2x^1 + 3x^2$

$$\frac{d}{dx}\ln(1-x) = -\frac{1}{1-x}$$

$$\frac{d}{dx}-\ln(1-x) = \frac{1}{1-x}$$

$$\ln(1-x) = -\int \frac{1}{1-x}\,dx + C$$

$$= -\int\left(\sum_{n=0}^{\infty} x^n\right)dx + C$$

$$= -\sum_{n=0}^{\infty}\left(\int x^n dx\right) + C$$

$$\ln(1-x) = -\sum_{n=0}^{\infty}\frac{1}{n+1}x^{n+1} + C$$

$$\ln(1-x) = -\sum_{n=1}^{\infty}\frac{1}{n}x^n + C = \sum_{n=1}^{\infty}-\frac{1}{n}x^n$$

$x=0$: $\ln(1) = 0 = \underbrace{-\sum_{n=1}^{\infty}\frac{1}{n}\cdot 0^n}_{0} + C$ $\qquad 0^n = 0$ IF $n \geq 1$

$$\boxed{0^0 = 1}$$

$C = 0$

HOW DOES DIFF. OR INT. A POWER SERIES
AFFECT THE INTERVAL OF CONVERGENCE?

THE RADIUS IS NOT AFFECTED BUT CONVERGENCE @ ENDPOINTS
OF INTERVAL <u>MAY</u> BE AFFECTED

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$$

$$\arctan x = \int \frac{1}{1-(-x^2)}\,dx + C$$

$$\vdots$$